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On Tensionless Strings in $3 + 1$ Dimensions

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Abstract

We argue for the existence of phase transitions in $3 + 1$ dimensions associated with the appearance of tensionless strings. The massless spectrum of this theory does not contain a graviton: it consists of one $N = 2$ vector multiplet and one linear multiplet, in agreement with the light-cone analysis of the Green-Schwarz string in $3 + 1$ dimensions. In M-theory the string decoupled from gravity arises when two 5-branes intersect over a three-dimensional hyperplane. The two 5-branes may be connected by a 2-brane, whose boundary becomes a tensionless string with $N = 2$ supersymmetry in $3 + 1$ dimensions. Non-critical strings on the intersection may also come from dynamical 5-branes intersecting the two 5-branes over a string and wrapped over a four-torus. The near-extremal entropy of the intersecting 5-branes is explained by the non-critical strings originating from the wrapped 5-branes.

1 Introduction

Tensionless strings have received much attention recently. They appear very naturally in six dimensions as non-trivial infrared fixed points. There are various infrared data which distinguish between different theories.

The six-dimensional tensionless string theory may carry either the $(0, 2)$ (minimal) or the $(0, 4)$ (which we also call $N = 2$) supersymmetry. The tensionless strings with $(0, 4)$ supersymmetry arise in compactification of type IIB on $K3$ [1] when the $K3$ gets an ADE singularity. The self-dual D3-branes of the type IIB theory wrap around the two cycles with small area producing the tensionless strings. A dual description of this is the M-theory on T^5/Z^2 [2, 3], where the tensionless strings arise when two or more parallel five branes coincide [4]. The connection between the two descriptions was found in [3].

Six-dimensional tensionless strings with $(0, 2)$ supersymmetry are found when a heterotic $E_8 \times E_8$ instanton shrinks to zero size [5]. The M-theory on S^1/Z_2 description of this effect is provided by a 5-brane attached to a boundary of spacetime (a 9-brane). More general examples of tensionless strings were constructed in [6]. For example, they arise at the strong coupling singularity of the heterotic string on $K3$ [6, 7].

In F-theory compactifications one finds tensionless strings when a two cycle shrinks to zero size [8, 9]. Its self-intersection number serves as a quantum number which distinguishes between different types of tensionless strings. In compactification of F-theory on Hirzebruch surfaces F_n the intersection number is $-n$. This gives $(0, 2)$ supersymmetry for $n \neq 2$, and $(0, 4)$ supersymmetry for $n = 2$. An interesting approach to solving the dynamics of tensionless strings using surface equations was proposed in [10].

In this paper we study tensionless strings in $3 + 1$ dimensions. Their existence in the context of F-theory was pointed out in [9]. If type IIB theory is compactified on a Calabi-Yau manifold, a tensionless string appears when a two-cycle shrinks to zero area with a 3-brane wrapped around it. From the $3 + 1$ dimensional point of view this tensionless string has $N = 2$ supersymmetry. In this paper we consider an M-theory description of such a theory, which is provided by two 5-branes intersecting over a 3-dimensional hyperplane. We show that the massless spectrum is in agreement with the light-cone analysis of the $3 + 1$ dimensional Green-Schwarz string (see section 3). Compactification to $2 + 1$ dimensions leads to a $N = 4$ abelian gauge theory which we study using D-brane methods in section 4. Going back to $3 + 1$ dimensions, we find an $N = 2$ linear multiplet interacting with strings. In section 5 we show that the near-extremal entropy of the intersecting 5-branes may be explained by a thermal ensemble of strings on the $3 + 1$

dimensional intersection.

2 From M-theory to Tensionless Strings

M-theory is the hypothetical unification of several types of 10-dimensional strings [11, 12]. Its low-energy effective description is the 11-dimensional supergravity, and some valuable information can be extracted from its classical solutions. The basic dynamical objects of the M-theory are the 2-brane, which is electrically charged under the 3-form gauge field, and its magnetic dual, the 5-brane. The dynamics governing the M-brane interactions is by no means well-understood. Some of its features may be inferred, however, from compactification to string theory, where the R-R charged p -branes have a remarkably simple description in terms of the Dirichlet (D-) branes [13–15].

The D-branes are objects on which the fundamental strings are allowed to end. There is evidence that a similar phenomenon takes place in M-theory: the fundamental 2-branes are allowed to have boundaries on the solitonic 5-branes [16, 4]. Thus, the 5-brane is the D-object of M-theory. The boundary of a 2-brane is a string, and the resulting boundary dynamics appears to reduce to a kind of string theory defined on the $5 + 1$ dimensional world volume. This picture has a number of interesting implications. Consider, for instance, two parallel 5-branes with a 2-brane stretched between them [4]. The two boundaries of the 2-brane give rise to two strings, lying within the first and second 5-branes respectively. The tension of these strings may be made arbitrarily small as the 5-branes are brought close together. In particular, it can be made much smaller than the Planck scale, which implies that the effective $5 + 1$ dimensional string theory is decoupled from gravity [1]. While it is not clear how to describe such a string theory in world sheet terms, it has been suggested that its spectrum is given by the Green-Schwarz approach [17] ($5 + 1$ is one of the dimensions where the Green-Schwarz string is classically consistent). In the limit of coincident 5-branes, we seem to find a theory of tensionless strings. These strings carry $(0, 4)$ supersymmetry in $5 + 1$ dimensions. Strings with $(0, 2)$ supersymmetry were explored from several different points of view in refs. [5–7].

In this note we propose that an M-theory phenomenon, similar to the one outlined above, leads to appearance of tensionless strings in $3 + 1$ dimensions. Here the relevant M-theory configuration involves two 5-branes intersecting over a 3-dimensional hyperplane. This is the coincident limit of the following 11-dimensional supergravity solution, constructed in refs. [18–20],

$$ds^2 = (F_1 F_2)^{-2/3} [F_1 F_2 (-dt^2 + dy_1^2 + dy_2^2 + dy_3^2)]$$

$$\begin{aligned}
& + F_1(dy_4^2 + dy_5^2) + F_2(dy_6^2 + dy_7^2) + \sum_{s=1}^3(dx_s)^2] \\
F_1^{-1} & = 1 + \frac{Q_1}{|\vec{x} - \vec{X}_1|}, \quad F_2^{-1} = 1 + \frac{Q_2}{|\vec{x} - \vec{X}_2|}, \\
\mathcal{F} & = 3(*dF_2^{-1} \wedge dy_4 \wedge dy_5 + *dF_1^{-1} \wedge dy_6 \wedge dy_7)
\end{aligned} \tag{2.1}$$

where \vec{X}_1 and \vec{X}_2 are the transverse positions of the 5-branes, and \mathcal{F} is the 4-form field strength. This solution describes a number of 5-branes (measured by Q_1) positioned in the (12345) hyperplane, and a number of 5-branes (measured by Q_2) positioned in the (12367) hyperplane.* This classical solution preserves 1/4 of the original 32 supersymmetries of the 11-dimensional supergravity [18–20].

Now consider a 2-brane stretched between the two 5-branes. Strings with the smallest possible tension result from the motion of the boundaries in the (123) hyperplane. These configurations are also special because they are supersymmetric. To maintain supersymmetry a straight 2-brane must intersect both 5-branes over a string, and this is only possible by positioning the 2-brane in a (αi) plane (the possible values of α are 1, 2, 3; the possible values of i are 8, 9, 10). The resulting configuration preserves 1/8 of the original 32 supercharges; i.e., the presence of a long straight string breaks $N = 2$ supersymmetry down to $N = 1$, from the $3 + 1$ dimensional point of view. This implies that the straight string is a BPS saturated state whose tension is protected by supersymmetry against quantum corrections.

The string tension is proportional to the transverse distance, $|\vec{X}_1 - \vec{X}_2|$. As this distance is made much smaller than the Planck length, we expect to find a $3 + 1$ dimensional string theory decoupled from gravity. Since this theory has 8 conserved supercharges, we find $N = 2$ supersymmetry in $3 + 1$ dimensions.[†] Remarkably, this is precisely the supersymmetry of the classically consistent Green-Schwarz superstring. In the next section we show that the Green-Schwarz construction indeed provides valuable information about the massless spectrum of this theory.

*For the most part we will chose $Q_1 = Q_2$ to correspond to a single 5-brane in each position.

[†]Note, for comparison, that the world volume theory of parallel 5-branes has 16 conserved supercharges, which corresponds to $N = 2$ in $5 + 1$ dimensions. Upon toroidal compactification to $3 + 1$, we find a theory with $N = 4$ supersymmetry.

3 Massless Modes of the Green-Schwarz string in $3 + 1$ dimensions.

In this section we carry out a naive light-cone quantization of the type II Green-Schwarz string. This theory is classically consistent in $D = 3, 4, 6, 10$, and has $N = 2$ supersymmetry in each of these cases. It is possible, therefore, that some features of the $D = 4$ model are relevant to the tensionless strings arising from the intersecting 5-branes of M-theory. In this section we examine the massless spectrum of the $D = 4$ model and find that, as expected, it contains no gravitons.

In the light-cone approach, the massless spectrum of the Green-Schwarz string is constructed out of the left- and right-moving fermion zero-modes, S_0 and \tilde{S}_0 . Each of these fields represents a massless Majorana fermion in $3 + 1$ dimensions. The transformation properties under the transverse rotation group, $SO(2)$, are labeled by the helicity. Each massless fermion has helicity $\pm 1/2$, so that the massless spectrum is constructed with the following 4 operators, $S_0^{\pm 1/2}$ and $\tilde{S}_0^{\pm 1/2}$.

As an exercise, let us first determine the spectrum of the type I string, which is constructed out of $S_0^{\pm 1/2}$ only. Clearly, there are only 4 states,

$$|0\rangle , \quad S_0^{\pm 1/2} |0\rangle , \quad S_0^{1/2} S_0^{-1/2} |0\rangle . \quad (3.1)$$

These states combine into two massless scalars and a Majorana fermion, which, as expected, form a $N = 1$ hypermultiplet.

In proceeding to the closed string case, we can again directly enumerate the states,

$$|0\rangle , \quad S_0^{\pm 1/2} |0\rangle , \quad \tilde{S}_0^{\pm 1/2} |0\rangle , \quad \text{etc.} . \quad (3.2)$$

We find that there are not enough degrees of freedom to form states of helicity ± 2 . As expected, there are no gravitons in the spectrum! Instead, we find states of helicity ranging from -1 to $+1$. Altogether, we have a vector field, 6 scalars, and 4 Majorana fermions. These states combine into one $N = 2$ $U(1)$ vector multiplet and one hypermultiplet.* It is interesting, that this is also the field content of a single $N = 4$ $U(1)$ multiplet. As explained above, the interactions will not respect the $N = 4$ supersymmetry, however. In the next section we will show that the massless spectrum found from the Green-Schwarz construction is in agreement with the counting of massless modes about the intersecting 5-branes in M-theory.

*A $N = 2$ hypermultiplet is isomorphic to a linear multiplet consisting of an antisymmetric tensor, B_{mn} , and 3 scalars.

One should be concerned about the lack of consistency of the $D = 4$ Green-Schwarz string at the quantum level. A similar lack of consistency of the $D = 6$ theory was addressed in ref. [17]. There it was suggested that extra world sheet degrees of freedom, half-integer moded oscillators, should be added to restore Lorentz invariance. This gives a consistent free string containing the twisted sector of the orbifold T^4/Z_2 . The absence of the untwisted sector makes it doubtful, however, that the interacting theory is consistent.

In principle, we could follow a similar procedure to obtain a Lorentz-invariant free string in $D = 4$. However, our primary interest here is in the massless spectrum, and the half-integer moded oscillators do not affect it. The naive light-cone procedure is sufficient to determine the massless spectrum which, as we will show, passes a number of consistency checks.

4 Tensionless Strings in 3 + 1 Dimensions

In this section we discuss the low-energy field theory description of the 5 \perp 5 configuration in M-theory. We argue that there exists a zero-energy bound state of the 5-branes, excitations above which are tensionless strings.

Compactification to the intersecting D4-branes.

First we appeal to a somewhat indirect method for obtaining information about the 5 \perp 5 configuration of M-theory: we compactify one of the coordinates of the intersection three-brane (say, y_1) on a circle. The double dimensional reduction of the M-theory 5-brane is known to produce a D4-brane of the type IIA theory. Therefore, upon compactification we find two D4-branes of type IIA theory positioned in the (2345) and (2367) hyperplanes. This is useful because a lot is known about the low-energy description of intersecting D-branes [21, 22]. More specifically, Sen [23] has found a solution of the 1 + 1 dimensional gauge theory describing two D3-branes intersecting over a string. Since this is related by T-duality to two D4-branes intersecting over a 2-brane, we will make some use of the results in [23].

When the branes do not quite intersect, it is still convenient to think of a pair of two-branes positioned in the (23) plane (as the transverse distance vanishes, these two 2-branes merge into a single intersection 2-brane). There is a natural Z_2 symmetry which exchanges the two 2-branes. The 2 + 1 dimensional theory which describes their world volume dynamics has 8 conserved supercharges; therefore, this is an $N = 4$ supersymmetric theory. When the 2-branes are separated along the transverse (8, 9 and 10) directions, their effective field theory description is given by the $N = 4$ $U(1) \times U(1)$ gauge theory

with two neutral hypermultiplets [21, 22]. A geometrical interpretation of these fields may be described as follows. The scalars of the first $U(1)$ multiplet describe the position of the first D4-brane in the transverse (8, 9 and 10) directions, while two of the scalars in the first neutral hypermultiplet describe its position in the 6 and 7 directions. Similarly, the second $U(1)$ multiplet describes the transverse position of the second D4-brane, while the second neutral hypermultiplet contains its 4 and 5 coordinates. It is quite clear that the neutral hypermultiplets do not participate in the dynamics because displacement of the D4-branes in the 4, 5, 6 and 7 directions has a trivial effect.

The only charged fields are in the massive hypermultiplet which contains two complex scalars with $U(1) \times U(1)$ charges $(1, -1)$ and $(-1, 1)$ (they correspond to boundaries of open strings stretched between the two D4 branes). It is convenient to combine the two $U(1)$'s into the vector (sum) and the axial (difference) combinations. The vector $U(1)$ corresponds to the overall motion of the two branes. Since all fields are neutral under the vector $U(1)$, it decouples. The axial $U(1)$ participates in the non-trivial dynamics because the two complex scalars carry charges ± 2 . As the transverse distance between the 4-branes is reduced, the charged states become lighter. Classically, they become massless at the point where the 4-branes intersect.

Thus, we are dealing with an interacting $N = 4$ field theory containing a $U(1)$ vector multiplet coupled to one charged hypermultiplet. This theory may be viewed as a dimensional reduction of the $N = 2$ supersymmetric theory in $3+1$ dimensions, which contains a $U(1)$ multiplet and a charged hypermultiplet [23]. Decomposing the content of the theory in terms of $N = 1$ multiplets, we have one $U(1)$ vector and three hypermultiplets, with charges 0, 2 and -2 . In terms of the charged $N = 1$ chiral superfields, Λ and $\bar{\Lambda}$, and the neutral chiral superfield, Φ , we have the superpotential

$$W_0 = \Phi \Lambda \bar{\Lambda} \tag{4.1}$$

Φ is related to the transverse separation between the D4-branes. When it vanishes in the classical theory, the charged fields become massless. The three scalars of the $D = 3$ vector multiplet have values in T^3/Z_2 ; the T^3 arises because the directions 8, 9 and 10 are compactified on a torus, while the Z_2 factor is due to the symmetry interchanging the two D4-branes. The $N = 4$ axial $U(1)$ multiplet also contains a vector field which, in $D = 3$, is dual to a compact scalar. This scalar also changes sign under the Z_2 , which implies that the classical moduli space on this Coulomb branch is T^4/Z_2 . Seiberg has argued that quantum effects turn the moduli space into a smooth $K3$ [24].

There is no Higgs branch which emanates from the classical singularity at the origin. This situation is similar to what happens in the case of parallel five branes, as discussed

in [6]. We will argue, however, that there exists a unique supersymmetric vacuum which describes a marginal bound state of the intersecting D4-branes. This bound state is related by T-duality to the marginal bound state of two D3-branes intersecting over a string which, in turn, is U-dual to winding states of the fundamental string [23]. Since marginal bound states are difficult to study, it is sometimes helpful to deform the problem in a way that turns it into a true bound state with a mass gap. The above superpotential does not allow for such a situation so we need to modify it next.

The modification is analogous to that used by Sen in the $1 + 1$ dimensional case [23]. He introduced a constant electric field, F_{01} . The necessity of screening this field in a supersymmetric vacuum drives the theory into the Higgs phase. Physically, the electric field corresponds to inserting some number of fundamental strings, which form a bound state together with the intersecting D3-branes.

What is the analogue of this phenomenon in $2 + 1$ dimensions? One possibility is to introduce a transverse magnetic field F_{12} . If we take its value to be independent of all the coordinates, then the parity is broken, but not the spatial translation invariance. To achieve a supersymmetric ground state, F_{12} needs to be screened. This creates an energy barrier against taking Φ to be large, since in that case the charges become arbitrarily massive. To determine the structure of the theory for small Φ , we follow [25, 23] and add mass terms to the superpotential,

$$W = \Phi\Lambda\bar{\Lambda} + \frac{1}{2}\epsilon\Phi^2 + \eta\Lambda\bar{\Lambda} \quad (4.2)$$

The non-trivial critical point of W , corresponding to a supersymmetric vacuum, is given by

$$\Phi = -\eta, \quad \Lambda = \bar{\Lambda} = \sqrt{\epsilon\eta} \quad (4.3)$$

This is a vacuum, where the magnetic field is screened. This phenomenon is nothing but the Meissner effect found in superconductors. In this phase we find that the magnetic flux is confined to the Abrikosov-Nielsen-Olesen strings. As we comment in the next section, these are likely to be the strings we are after.

There seems to exist another deformation of the theory that turns the marginal bound state into a state with a mass gap. We may add the abelian Chern-Simons term for the gauge field which will cause the vector multiplet to become massive (its coefficient is the mass of the $U(1)$ multiplet).

The $3 + 1$ -dimensional theory on the $5 \perp 5$ intersection

From the results above, we have some information about what happens to the $3 + 1$ dimensional intersection theory upon compactification on a circle to $2 + 1$ dimensions.

Now we attempt to “undo” the compactification and learn more about the exotic theory of tensionless strings. Indeed, it is not hard to see how the degrees of freedom in the $2 + 1$ dimensional theory combine into the $N = 2$ multiplets of the $3 + 1$ dimensional theory. When the 5-branes are separated, we have $N = 2$ supersymmetric $U(1) \times U(1)$ theory with two neutral linear multiplets. Note that each of the branes contributes a $N = 2$ vector multiplet and a linear multiplet, precisely in accord with the spectrum of the $3 + 1$ dimensional Green-Schwarz string discussed in section 3. Now it is natural to interpret the scalars from the vector multiplets as corresponding to motion in the 4, 5, 6 and 7 directions (upon dimensional reduction, the vector multiplets reduce to the duals of the neutral hypermultiplets in $2 + 1$ dimensions). Each $N = 2$ linear multiplet contains B_{mn} and 3 scalars. The antisymmetric tensor, which is dual to a compact scalar (axion), originates from the dimensional reduction of a $D = 6$ antiselfdual two-form field, B_{mn}^- . The 3 scalars in the first (second) linear multiplet describe the transverse (8, 9 and 10) coordinates of the first (second) 5-brane. Combining the two linear multiplets into a sum and a difference, we see that the sum corresponds to the overall transverse motion. Only the difference multiplet participates in the dynamics.

The crucial question is: what is the $3 + 1$ dimensional realization of the extra charged multiplets, which become massless in $2 + 1$ dimensions in the limit when the two branes coincide. Since in $2 + 1$ dimensions we found one $N = 4$ hypermultiplet containing charges $(-1, 1)$ and $(1, -1)$ under $U(1) \times U(1)$, it is clear that in $3 + 1$ dimensions we have a multiplet containing axionic charges $(-1, 1)$ and $(1, -1)$ under the two linear multiplets. Classically, these charges are carried by long straight strings. As the strings become tensionless, their winding modes turn into the massless charges of the $2 + 1$ dimensional compactified theory. There are two types of tensionless strings which come from the two possible orientations of the 2-brane stretched between the 5-branes. There is also a Z_2 action which exchanges the two fivebranes.

The classical moduli space of the difference linear multiplet is parameterized by four real compact scalars. Three of the scalars denote the (8, 9, 10) relative motion and change sign under the Z_2 . The other compact scalar is dual to the B_{mn} field and also changes sign under the Z_2 . So, the classical moduli space is T^4/Z_2 . Classically, we find tensionless strings at the singularity of this moduli space. As for the $2 + 1$ dimensional system analyzed above, we expect no Higgs branch to emanate from the singularity point. We believe, however, that there exists a supersymmetric vacuum corresponding to a marginal bound state of the intersecting 5-branes.

In the previous section we argued that the problem may be deformed in such a way that the marginal bound state turns into a state with a mass gap. This bound state

is described by the Higgs phase of a supersymmetric Landau-Ginzburg model in $2 + 1$ dimensions where the charged fields have condensed.* It is well-known that this model contains string-like solitons, the Nielsen-Olesen vortices. Let us recall their basic structure. We will work with a $N = 2$ $U(1)$ multiplet in $2 + 1$ dimensions interacting with a charged hypermultiplet. In the Higgs phase, the charged scalars acquire expectation values, and the massless $U(1)$ vector multiplet combines with the hypermultiplet to form a massive vector multiplet. The BPS saturated Nielsen-Olesen strings arise from the interaction of a charged scalar with the $U(1)$ field. The static energy of a string of length L is given by

$$E = L \int d^2x \left[\frac{1}{4e^2} F_{mn}^2 + |D_m \Lambda|^2 + \lambda \left(|\Lambda|^2 - \frac{F^2}{2} \right)^2 \right] \quad (4.4)$$

$F = \sqrt{\epsilon\eta}$ is the expectation value of the charged scalar, Λ . If we chose $2\lambda = e^2$, so that both the scalar mass and the vector mass are equal to $m = eF$, then there exists a Bogomolny bound on the energy of the vortex solution [26]. If the vortex contains n units of the magnetic flux, then

$$E \geq F^2 L \pi |n| \quad (4.5)$$

When the bound is saturated, the vortex solution breaks only $1/2$ of the supersymmetries. The elementary vortices of charge ± 1 are presumably related to the strings that arise at the M-brane intersections.

In order to see which objects govern the low-energy dynamics of the abelian Higgs model, we have to compare the value of the Nielsen-Olesen string tension to the scalar and vector mass, m . From the above equations, we find

$$T = \frac{m^2 \pi}{e^2} \quad (4.6)$$

For small e (weak coupling) the strings are heavy and the model has a conventional field theoretic description. This is the usual limit in which the abelian Higgs model is discussed. Note, however, that for strong coupling the strings should become more relevant because their tension becomes small.[†] Here we may find a non-field theoretic behavior dominated by the tensionless strings. It is tempting to speculate that, as the theory flows to strong coupling, the tensionless strings regulate the Landau pole found in the purely field theoretic approach.

We believe that the Nielsen-Olesen strings that become light at strong coupling provide a good model for the tensionless strings that are found in M-theory when the 5-branes

*In $3 + 1$ dimensions this should look like condensation of strings.

[†]We are very grateful to J. Distler for illuminating discussions of this point.

approach each other. Clearly, we need a better understanding of how this comes about at the quantum level, but we find it remarkable that the M-theory may provide a quantum description of fluctuating superconducting vortex lines.

In conclusion we would like to mention a different field-theoretic scenario for the transition that takes place when the 5-branes coincide. The interacting system relevant to the separated 5-branes consists of a linear multiplet coupled to strings whose tension is proportional to the separation. We may imagine that these strings are the Nielsen-Olesen strings discussed above. In other words, the strings originate from a hidden $N = 2$ supersymmetric $U(1)$ gauge theory coupled to a charged hypermultiplet in the Higgs phase. The expectation value of the linear multiplet is identified with the separation between the 5-branes.

We denote the scalars in the linear multiplet as a complex field θ and a real field r . They transform in the adjoint representation of the $SU(2)_R$ symmetry. There is a coupling of these scalars to the D-terms of the gauge field which also transform under the adjoint of $SU(2)_R$. By supersymmetry there is also a coupling of the B field to the field strength of the gauge field of the form $B_{mn}F^{mn}$. As a result, the D-term constraints are

$$\Lambda\bar{\Lambda} - \theta = 0, \quad |\Lambda|^2 - |\bar{\Lambda}|^2 = r. \quad (4.7)$$

A solution exists for any value of r and θ . In particular, $\Lambda = \bar{\Lambda} = 0$ if and only if $r = \theta = 0$: namely, when the two five-branes coincide.

As the 5-branes coincide, the hidden $U(1)$ becomes “un-Higgsed” and it seems that on the other side of the phase transition we have a Coulomb phase of $U(1)$ coupled to a charged hypermultiplet. It is also possible that instead of the Coulomb phase we have a single quantum state describing the bound 5-branes.

5 The Near-Extremal Entropy

In this section we provide another piece of evidence for the existence of a string theory on the $3 + 1$ dimensional intersection of two M-theory 5-branes. This evidence, albeit indirect, comes from studying the near-extremal entropy of the intersecting 5-branes. The necessary non-extremal supergravity solution was recently found by Cvetic and Tseytlin [27], and we will simply use their results.

The solution is characterized by the non-extremality parameter μ , and the charges Q_1 and Q_2 which are proportional to the numbers of the (12345) and (12367) 5-branes

respectively. For small μ the ADM mass and the Bekenstein-Hawking entropy are [27]

$$\begin{aligned} M &= b(Q_1 + Q_2 + \mu + O(\mu^2)) \\ S_{BH} &= c\mu\sqrt{Q_1Q_2} + O(\mu^2) \end{aligned} \quad (5.1)$$

Using $c/b = 4\pi$, we have near extremality,

$$S_{BH} = 4\pi\sqrt{Q_1Q_2}E, \quad (5.2)$$

where $E = M - M_0$. This implies that the Hawking temperature is constant and does not depend on E ,

$$T_H = \frac{1}{4\pi\sqrt{Q_1Q_2}}. \quad (5.3)$$

A similar phenomenon also occurs for a 5-brane in 10 dimensions and was interpreted by Maldacena [28] as due to a gas of strings at its Hagedorn temperature, which is equal to the Hawking temperature.

Consider a gas of strings with tension $T_{eff} = 1/(2\pi\alpha'_{eff})$, and with world sheet degrees of freedom having central charge c_{eff} . At high energy E , the entropy is the same as for a single long string,

$$S = 2\pi\sqrt{\frac{c_{eff}\alpha'_{eff}}{6}}E \quad (5.4)$$

Comparing with the Bekenstein-Hawking entropy, we interpret the near-extremal entropy in terms of a single string with

$$c_{eff}\alpha'_{eff} = 24Q_1Q_2 \quad (5.5)$$

Now we need to recall how Q_1 and Q_2 are related to n_1 , the number of 5-branes in the (12345) plane, and n_2 , the number of 5-branes in the (12367) plane. This was explained in [29], with the result

$$Q_1 = \frac{n_1}{2\pi L_6 L_7} \left(\frac{\pi\kappa^2}{2} \right)^{1/3}, \quad Q_2 = \frac{n_2}{2\pi L_4 L_5} \left(\frac{\pi\kappa^2}{2} \right)^{1/3}. \quad (5.6)$$

Here L_i are the sizes of the compact dimensions, (4, 5, 6, 7), and κ is the 11-dimensional gravitational constant.

We will assume that c_{eff} does not depend on the parameters, while the effective tension does. This leads to

$$T_{eff} \sim \frac{L_4 L_5 L_6 L_7}{n_1 n_2 \kappa^{4/3}}. \quad (5.7)$$

In other words, the effective tension is proportional to the volume of the (4, 5, 6, 7) dimensions. J. Maldacena suggested to us an interesting interpretation of our formula for the

string tension: the string is the 5-brane wrapped around the $(4, 5, 6, 7)$ dimensions. This interpretation passes some consistency checks. For instance, if $n_1 = n_2 = 1$ we expect the string to originate from a single 5-brane. Using the known value of the 5-brane tension [29],

$$T_5 = \left(\frac{\pi}{2}\right)^{1/3} \kappa^{-4/3}, \quad (5.8)$$

we find that $T_{eff} = T_5 L_4 L_5 L_6 L_7$ is consistent with (5.5) provided we use $c_{eff} = 6$. For general n_1 and n_2 , the effective string tension is

$$T_{eff} = \frac{T_5 L_4 L_5 L_6 L_7}{n_1 n_2} \quad (5.9)$$

Thus, the tension of a wrapped 5-brane is reduced by a multiplicative factor. A similar reduction of tension was necessary for explaining the entropy of a D-string moving within a number of parallel type IIB 5-branes [28].

Thus, the non-critical strings relevant to the $5\perp 5$ configuration with no transverse separation are M-theory 5-branes wrapped around T^4 . This may seem surprising, since for transversely separated 5-branes the strings originate from stretched 2-branes. When the 5-branes are coincident, and the transverse coordinates $(8, 9, 10)$ are compact, we may, in fact, add a 2-brane intersecting the 5-branes and wrapped around one of the transverse dimensions (say, 8). This would create a string in $3 + 1$ dimensions with tension $T_2 L_8 / (n_1 n_2)$.^{*} However, for the classical solution (2.1) the transverse coordinates are non-compact, and this is impossible.

Let us recall that there exists another configuration of M-theory which preserves $1/8$ of the supersymmetries [18–20]: we can add a 5-brane in the $(a4567)$ hyperplane, with $a = 1, 2$ or 3 . Thus, the additional 5-brane has a string in common with the (12345) and (12367) hyperplanes. The low-energy excitations of the additional 5-brane will make the string fluctuate within the (123) hyperplane, while its tension is given by (5.9). This provides a fairly consistent picture of the $3 + 1$ dimensional non-critical string which arises for strictly intersecting 5-branes. However, it remains to be explained precisely why this string has $c_{eff} = 6$.

If the volume of T^4 , $L_4 L_5 L_6 L_7$, is made very small in Planck units, the string seems to decouple from 11-dimensional gravity. Increasing n_1 and n_2 also serves to reduce

^{*}A similar configuration of M-theory explains the effective string picture of [28]. Consider a 5-brane solution with one of the transverse dimensions taken to be compact. We may then add a 2-brane intersecting the 5-brane over a string and wrapped around the compact dimension. Upon reduction to type IIA theory, this gives a fundamental string moving within the NS-NS 5-brane. This is U-dual to the D-string moving within the D5-brane, which was studied in [28].

the effective tension of the $3 + 1$ dimensional string. To summarize, in this section we interpreted the near-extremal entropy of the $5 \perp 5$ configuration in terms of strings on the $3 + 1$ dimensional intersection. The parameters may be adjusted in such a way that this string theory decouples from gravity, as anticipated from our previous analysis.

6 Discussion

In this paper we have argued that tensionless strings are not unique to $5+1$ dimensions. A concrete $3 + 1$ -dimensional example is provided by the $5 \perp 5$ configuration of M-theory. The two 5-branes may be connected by a 2-brane, whose boundary then acts as a non-critical string. Another source of non-critical strings is a 5-brane which intersects the two original 5-branes over a string and is wrapped over T^4 . The tension of the non-critical strings may be made arbitrarily small. Thus, the graviton should not be a part of the spectrum. Indeed, we find that the only massless fields the string couples to lie in the vector and hyper $N = 2$ multiplets. This agrees with the light-cone analysis of the massless spectrum of the Green-Schwarz superstring in $3 + 1$ dimensions.

We would also like to remark that tensionless strings may be found in $4 + 1$ dimensions, simply by dimensionally reducing the $5 + 1$ dimensional case. This case has a direct D-brane description. Indeed, consider the M-theory configuration consisting of two parallel 5-branes connected by a 2-brane. Double dimensional reduction of the 5-branes along a direction orthogonal to the 2-brane brings us to the type IIA configuration of two parallel D4-branes connected by a stretched D2-brane. This configuration is described by a $SU(2)$ monopole in the world volume $N = 4$ gauge theory. To see this, note that T-duality relates it to a pair of D3-branes connected by a D1-brane. The relation of this to monopoles was explained in [30–32]. Far away from the monopole the Higgs fields reaches a constant value which measures the separation between the 3-branes; at the core of the monopole, the Higgs field vanishes which means that the 3-branes are connected by a D-string. Clearly, the monopole may also be regarded as a special solution of the $4 + 1$ dimensional gauge theory, which is translationally invariant along one of the directions. This solution describes two parallel D4-branes connected by a D2-brane. Presumably, there also exists a gauge theory configuration describing a 2-brane stretched between two intersecting D4-branes, but we leave this for future work.

In conclusion we would like to comment on the relation between supersymmetric non-critical strings and gauge theories. It has been proposed that, upon compactification on T_2 , the anti-selfdual string in $D = 6$ reduces to $N = 4$ supersymmetric $SU(n)$ gauge theory in $D = 4$ [1, 17]. The electric and magnetic charges in $D = 4$ arise as strings

wrapped around the two different cycles of T_2 .

There exists a similar relation between $N = 2$ supersymmetric non-critical strings in $D = 4$ and $N = 4$ gauge theory in $D = 3$ obtained by compactification on a circle. If we start in the M-theory with n 5-branes in the (12345) plane and m 5-branes in the (12367) plane, then upon dimensional reduction we arrive at n D4-branes intersecting m D4-branes. The intersection is described by $D = 3$ $N = 4$ supersymmetric $U(1) \times SU(n) \times SU(m)$ gauge theory coupled to three hypermultiplets [21, 23]. Two of the hypermultiplets are neutral under $U(1)$; the first is in the adjoint representation of $SU(n)$, the second – of $SU(m)$. The third hypermultiplet contains a (n, \bar{m}) representation of $U(1)$ charge 2, and a (\bar{n}, m) representation of $U(1)$ charge –2. In this paper we analyzed the purely abelian case of $n = m = 1$. The non-abelian dynamics is more complicated, and we hope to return to it in the future.

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References

- [1] E. Witten, “Some Comments On String Dynamics,” hep-th/9507121.
- [2] K. Dasgupta and S. Mukhi, “Orbifolds of M-Theory,” hep-th/9512196.
- [3] E. Witten, “Five-branes And M-Theory On An Orbifold,” hep-th/9512219.
- [4] A. Strominger, “Open P-Branes,” hep-th/9512059.
- [5] O. Ganor and A. Hanany, “Small E_8 Instantons and Tensionless Non-critical Strings,” hep-th/9602120.
- [6] N. Seiberg and E. Witten, “Comments on String Dynamics in Six Dimensions,” hep-th/9603003.
- [7] M. Duff, H Lu and C. Pope, “Heterotic Phase Transitions and Singularities of the Gauge Dyonic String,” hep-th/9603037.

- [8] D. Morrison and C. Vafa, “Compactifications of F-Theory on Calabi-Yau Threefolds, I and II,” hep-th/9602114, 9603161.
- [9] E. Witten, “Phase Transitions In M-Theory And F-Theory,” hep-th/9603150.
- [10] O. Ganor, “Six Dimensional Tensionless Strings in the Large N Limit,” hep-th/9605201.
- [11] C. M. Hull and P. K. Townsend, “Unity of Superstring Dualities,” hep-th/9410167, *Nucl. Phys.* **B438** (1995) 109.
- [12] E. Witten, “String Theory Dynamics In Various Dimensions,” hep-th/9503124, *Nucl. Phys.* **B443** (1995) 85.
- [13] J. Dai, R. G. Leigh and J. Polchinski, “New Connections Between String Theories,” *Mod. Phys. Lett.* **A4** (1989) 2073.
- [14] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges,” hep-th/9510017.
- [15] For a recent review, see J. Polchinski, S. Chaudhuri, and C. V. Johnson, “Notes on D-Branes,” NSF-ITP-96-003, hep-th/9602052.
- [16] P. K. Townsend, “D-branes from M-branes,” hep-th/9512062.
- [17] J. Schwarz, “Self-Dual Superstring in Six Dimensions,” hep-th/9604171.
- [18] G. Papadopoulos and P. K. Townsend, “Intersecting M-branes,” hep-th/9603087.
- [19] A. A. Tseytlin, “Harmonic superpositions of M-branes,” hep-th/9604035.
- [20] J. P. Gauntlett, D. A. Kastor and J. Traschen, “Overlapping Branes in M-Theory,” hep-th/9604179.
- [21] M. Bershadsky, V. Sadov and C. Vafa, “D-Strings on D-Manifolds,” hep-th/9510225.
- [22] M. Green, J. A. Harvey and G. Moore, “I-Brane Inflow and Anomalous Couplings on D-Branes,” hep-th/9605033.
- [23] A. Sen, “U-Duality and Intersecting D-Branes,” hep-th/9511026.
- [24] N. Seiberg, “IR Dynamics on Branes and Space-Time Geometry,” hep-th/9606017.
- [25] E. Witten, “Bound States Of Strings And p -Branes,” hep-th/9510135.
- [26] E. Bogomolny, “Stability of Classical Solutions,” *Yadernaya Fizika* **24** (1976) 861.
- [27] M. Cvetic and A. Tseytlin, “Non-Extreme Black Holes from Non-Extreme Intersecting M-branes,” hep-th/9606033.
- [28] J. Maldacena, “Statistical Entropy of Near Extremal Five-Branes,” hep-th/9605016.
- [29] I. R. Klebanov and A. Tseytlin, “Four-Dimensional Black Holes as Intersecting M-branes,” hep-th/9604166.
- [30] A. A. Tseytlin, “Self-duality of Born-Infeld action and Dirichlet 3-brane of type IIB superstring theory,” hep-th/9602064.

- [31] M. Green and M. Gutperle, “Comments on Three-Branes,” hep-th/9602077.
- [32] M. Douglas and M. Li, “D-Brane Realization of N=2 Super Yang-Mills Theory in Four Dimensions,” hep-th/9604041.